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RECONSTRUCTION OF BED PRESSURE WITH DATA OF NORMAL WELL OPERATION

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The article suggests an algorithm for reconstructing the bed pressure from measurements of pressure and yields of wells in the course of their normal operation.

In solving the problem of developing petroleum deposits it becomes necessary to check changes of pressure in the oil bearing bed. The existing methods consist in estimating the bed pressure, i.e., the pressure that would establish itself at the bottom of a shutdown well after complete stoppage of inflow of liquid. At present this value is determined chiefly by a depth manometer in the course of unsteady investigations involving shutdown of the well. In practice, it is often impossible or undesirable to shut down an operating well. Methods are therefore worked out which make it possible to estimate the bed pressure from data of normal well operation.

All the known methods of estimating bed pressure are in essence methods of identificationary nature, and they are not suitable for reconstructing the pressure distribution near an operating well. The lack of this kind of information complicates the mathematical modeling of the processes of oil extraction in the sense that it prevents the solution of nonsteady filtration in the traditional initial and boundary statement. Without an operative estimate of the pressure distribution in the bed it is difficult to state the initial conditions because for that it is necessary to solve successively a considerable number of normal problems where the result of the preceding problem is taken as the initial condition for each problem upon change of the operating regime.

In the present work the problem is reduced to restoring the initial condition (and thus also the subsequent states) for an equation of parabolic type from redefined boundary conditions. It is assumed that the time of piezoconductivity and the hydroconductivity of the bed are specified (they may be, e.g., determined previously in the course of nonsteady investigations of wells [1]). The stated problem belongs to the class of inverse retrospective problems, and as is well known, it is a malposed problem. It is solved by the method of ordered minimization of the mean risk [2]; this makes it possible under conditions of the static approach to obtain guaranteed solutions (with a certain probability) from a limited amount of empirical data.

Nonsteady filtration of petroleum in a well (s = 1) or in a gallery of wells (s = 0) is described by the equation

$$\frac{\partial p}{\partial t} = Lp \equiv \frac{1}{x^s} \frac{\partial}{\partial x} \left(x^s \frac{\partial p}{\partial x} \right),\ x_0 < x < 1,\ 0 < t < \infty, \tag{1}$$

$$p(x,\ 0) = \varphi(x), \tag{2}$$

$$p(x_0,\ t) = \psi(t), \tag{3}$$

$$\frac{\partial p(1,\ t)}{\partial x} = 0, \tag{4}$$

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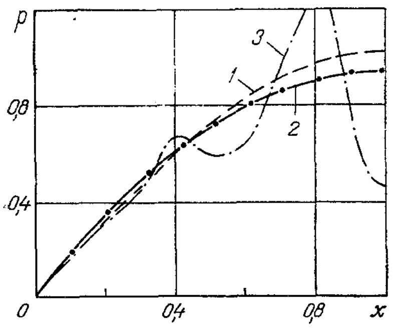

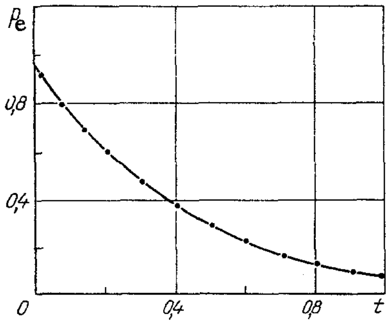

Fig. 1

Fig. 2

Fig. 1. Estimates of the initial pressure distribution: 1) n = 1; 2) n = 2; 3) n = 6. Dots: function $\phi(x)$.

Fig. 2. Graph of the function $p_e(1, t)$ for $n_* = 2$. Dots: true pressure at the point x = 1.

The form of the boundary condition with x = 1 depends on the regime of developing the petroleum deposit. For the sake of determinacy this condition was taken in the form (4) which corresponds to the working of a bed to exhaustion.

We assume that within some time interval $0 < T_1 \leq t \leq T_2 < \infty$ the oil yield is measured

$$q(t) = \frac{\partial p(x_0, t)}{\partial x}, \quad t \in [T_1;\ T_2]. \tag{5}$$

This poses the retrospective inverse problem of determining the initial pressure distribution $\phi(x)$ from the boundary conditions (3)-(5).

The solution is sought in the form

$$\varphi(x) = \sum_{m=1}^{\infty} a_m X_m(x) + \psi(0), \tag{6}$$

where $X_m(x)$ are the eigenfunctions of the problem

$$LX_m = -\lambda_m^2 X_m, \quad X_m(x_0) = 0, \quad X_m'(1) = 0.$$

The solution of the primal problem (1)-(4) is written as

$$p(x, t) = \sum_{m=1}^{\infty} \{[a_m \exp(-\lambda_m^2 t) + b_m F_m(t)] X_m\} + \psi(t), \tag{7}$$

where

$$b_m = -\frac{\int_{x_0}^{1} X_m(x)\,dx}{\int_{x_0}^{1} X_m^2(x)\,dx};$$

$$F_m(t) = \psi(t) - \psi(0)\exp(-\lambda_m^2 t) - \lambda_m^2 \int_0^t \psi(\tau)\exp[-\lambda_m^2(t-\tau)]\,d\tau.$$

Condition (5) assumes the form

$$\tilde{q}(t) \equiv q(t) - \sum_{m=1}^{\infty} b_m X_m'(x_0) F_m(t) = \sum_{m=1}^{\infty} a_m X_m'(x_0) \exp(-\lambda_m^2 t), \tag{8}$$

$$t \in [T_1;\ T_2].$$

In accordance with (6), determination of the initial bed pressure reduces to finding the coefficient a_m from condition (8). The uniqueness of the solution of this problem follows from the uniqueness of the expansion of the function $\tilde{q}(t)$ into a Dirichlet series [3].

We represent the measurements of the yield in the form of a sample

$$\tilde{q}_e(t_k) = \tilde{q}(t_k) + \varepsilon_k, \quad t_k \in [T_1;\ T_2], \quad k = 1, 2, \ldots, l, \tag{9}$$

TABLE 1. Results of Calculations by the Method of Ordered Minimization of the Mean Risk

n	$\tilde{c}$	$I_0(n)\cdot 10^2$	Ω^{-1}	$I(n)\cdot 10^2$
1	$\tilde{c}_1 = 1{,}044$	0,088	0,409	0,189
2	$\tilde{c}_1 = 0{,}994$ $\tilde{c}_2 = 0{,}046$	0,001	0,307	0,005
3	$\tilde{c}_1 = 0{,}996$ $\tilde{c}_2 = 0{,}046$ $\tilde{c}_3 = 0{,}001$	0,001	0,236	0,007
4	$\tilde{c}_1 = 0{,}994$ $\tilde{c}_2 = 0{,}046$ $\tilde{c}_3 = -0{,}001$ $\tilde{c}_4 = 0{,}001$	0,001	0,180	0,010
6	$\tilde{c}_1 = 0{,}994$ $\tilde{c}_2 = 0{,}052$ $\tilde{c}_3 = -0{,}037$ $\tilde{c}_4 = 0{,}130$ $\tilde{c}_5 = -0{,}198$ $\tilde{c}_6 = 0{,}095$	0,001	0,099	0,018

where ε_k are the stochastic errors of measurement with zero mathematical expectation and finite dispersion.

Then the initial pressure distribution can be represented by the finite sum

$$\varphi_e(x) = \sum_{i=1}^{n} \tilde{c}_i X_i(x) + \psi(0), \tag{10}$$

where

$$\tilde{c} = \operatorname{arginf} I_0(c);\; I_0(c) = \frac{1}{l}\sum_{k=1}^{l} \xi_k^2;\; \xi_k = \tilde{q}_э(t_k) - \sum_{i=1}^{n} B_{ki} c_i;$$

$$B_{ki} = X_i'(x_0)\exp(-\lambda_i^2 t_k),\; i = 1, 2, \ldots, n,\; k = 1, 2, \ldots, l.$$

Regularization of the malposed problem is attained by choosing the number of terms of the sum n in (10) in accordance with the procedure of ordered minimization of the mean risk. It was shown [2] that for each n it is possible with the probability $1 - \eta$ to construct the upper estimate of the mean risk

$$I(n) = I_0(\tilde{c})\,\Omega\left(\frac{l}{n};\; \frac{\ln\eta}{n}\right). \tag{11}$$

In this expression the magnitude of I_0 determines the error of approximating the function $\tilde{q}(t)$ by a finite sum, and the factor Ω determines the degree to which the complexity of the model (magnitude of n) corresponds to the size of the sample ℓ. The first factor in (11) decreases with increasing n, the second factor increases. The method of ordered minimization of the risk consists in the following: We have to find $n = n_*$ minimizing the estimate (11), and take the function (10) with $n = n_*$ as the solution of the inverse problem. It was proved [2] that with increasing amount of empirical data the sequence of solutions obtained by the method of ordered minimization of the risk converges to the sought solution.

We note that within the framework of the procedure of regularization after A. N. Tikhonov, the solution may also be represented in the form of a series of eigenfunctions [4, 5]. The advantage of the method of structural minimization of the mean risk used by us lies in its simplicity, and also in the fact that for the selection of the optimum number of terms of the series by this method it is not necessary to know the magnitude of the experimental error, whereas when the principle of discrepancy is used, information on the experimental error is indispensable.

For practical calculations we use in our work the estimate [2]

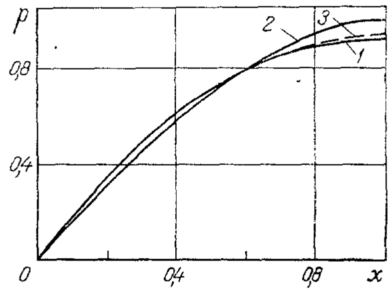

Fig. 3. Estimates of the initial pressure distribution: 1) relative error 10%; 2) 20%; 3) true pressure distribution with t = 0.

$$I(n) = \left[\frac{I_0(\tilde{c})}{1 - \sqrt{\frac{n\,[\ln(l/n) + 1] - \ln\eta}{l}}} \right]_\infty, \quad (12)$$

where

$$[z]_\infty = \begin{cases} z, & z \geq 0, \\ \infty, & z < 0. \end{cases}$$

As model problem we will examine the start of a gallery of wells at the instant $t = -t_0$. Let $p(x, -t_0) = 1$, $p(x_0, 0) = 0$. In that case $s = 0$, $x_0 = 0$, $X_m(x) = \sin\lambda_m X$, $\lambda_m = \pi(m - 1/2)$,

$$\varphi(x) = p(x, 0) = \sum_{m=1}^{\infty} a_m \sin\lambda_m x, \quad (13)$$

where $a_m = 2\exp(-\lambda_m^2 t_0)/\lambda_m$.

From the solution of the primal problem we determine the yield q(t), and by the noise of this function we determine the "measurements" of the yield

$$\tilde{q}_e(t_k) = 2\sum_{m=1}^{\infty} \exp[-\lambda_m^2(t_k + t_0)] + \varepsilon_k. \quad (14)$$

The pressure distribution with $t \geq 0$ is estimated by the formula

$$p_e(x, t) = \sum_{i=1}^{n_*} \tilde{c}_i \exp(-\lambda_i^2 t) \sin\lambda_i x, \quad (15)$$

where the values of n_* and $\tilde{c}_i$ are found by the ordered minimization of the estimate (12). The results of the calculations for $t_0 = 0.1$; $t_k = 0.01\,k$, $k = 1, 2, \ldots, 20$; $n = 1, 2, 3, 4, 6$ and of the "noise" modeling the relative error of measurements of 1% are presented in Table 1. For comparison we present the accurate values of the first three coefficients of the series (13): $a_1 = 0.9948$; $a_2 = 0.0461$; $a_3 = 0.0005$. It can be seen from Table 1 that I attains its minimum value for $n_* = 2$.

Figure 1 illustrates lucidly the loss of stability when the complexity of the model increases. It can be seen from Fig. 2 that restoration of the initial pressure makes it possible to estimate correctly the change of bed pressure for $t \geq 0$.

Calculations were also carried out in modeling a relative error of 10 and 20%. Figure 3 presents the estimates of $\phi_e(x)$ thereby obtained. It can be seen that the method is stable in regard to changes of the level of the experimental errors.

NOTATION

p, pressure; x and t, dimensionless spatial and temporal variables, respectively; q, petroleum yield; ℓ, number of measurements of the yield; x_0, dimensionless well radius.

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LONGITUDINAL RADIATION TRANSFER IN MOVEMENT OF A DUST CONTAINING MEDIUM IN A PLANE CHANNEL

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The article presents the solution of the problem of radiative and convective heat exchange in stabilized flow of a scattering medium in a plane channel, with the longitudinal radiative fluxes taken into account.

In radiative and convective heat exchange in short channels [1], and also in channels with abrupt changes of the wall temperature [2, 3], the transfer of radiation along the flow of the medium is considerable. For instance, in practice, the problem of shielding the radiation of the hot inlet end face of a channel from its cooling walls is encountered. To determine the effectiveness of shielding, experiments are needed, but also mathematical modeling of the radiative and convective heat exchange.

Radiative and convective heat exchange in a circular cylindrical channel with a view to the longitudinal radiative fluxes was dealt with in [1-6]. The authors assumed that the flux was hydrodynamically stabilized, and that the flow was both laminar [1-5] and turbulent [1-6]. The authors of [4, 6] took the selectivity of the optical properties of the medium into account, the authors of [5] took scattering of the radiation into account. For a plane channel the problem was solved in [1], and it was assumed there that the medium is nonscattering and the flow laminar.

Let us examine heat exchange in stabilized flow of a radiating, absorbing, anisotropically scattering, heat conducting medium in a plane channel with black walls (Fig. 1). We regard the medium as nonselective. We neglect heat conduction along the flow. Mathematically the problem is formulated in the form of an equation of energy

$$\varphi \frac{\partial \theta}{\partial \bar{x}} = \frac{2}{\mathrm{Pe}} \frac{\partial}{\partial \bar{y}} A \frac{\partial \theta}{\partial \bar{y}} - \frac{H}{\rho c_p \langle v \rangle T_0} \operatorname{div} \mathbf{q}_R \quad (1)$$

with the boundary conditions

$$\theta(\bar{y}) = 1, \quad \bar{x} = 0, \quad (2)$$

$$\theta(\bar{y} = 0) = \theta(\bar{y} = 1) = \theta_1, \ 0 < \bar{x} \leqslant \bar{a}, \quad (3)$$

$$\theta(\bar{y} = 0) = \theta(\bar{y} = 1) = \theta_2, \ \bar{a} < \bar{x} < \bar{L}, \quad (4)$$

$$\theta(\bar{y}) = \theta_2, \ \bar{x} = \bar{L}. \quad (5)$$

The following notation was used: $\theta = T/T_0$, $\theta_1 = T_1/T_0$, $\theta_2 = T_2/T_0$, $\bar{x} = x/H$, $\bar{y} = y/H$, $\bar{a} = a/H$, $\bar{L} = L/H$, $\phi = v/\langle v \rangle$, $\mathrm{Pe} = 2H\rho c_p \langle v \rangle/\lambda$, $A = 1 + \lambda_S/\lambda$.

The expression for the divergence of the flow density vector of the radiation has the form [7]

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